El extraño problema de la información cuántica y el comportamiento de los agentes *(Agreement between Observers)*

Giannicola Scarpa

Universidad Politécnica de Madrid, Spain

Joint work: Patricia Contreras-Tejada (ICMAT), Aleksander M. Kubicki (UCM), Adam Brandenburger (NYU) and Pierfrancesco La Mura (HHL Leipzig)

Nat Commun 12, 7021 (2021)

UNIVERSIDAD POLITÉCNICA DE MADRID

Is the world quantum?

It is a well-known fact that quantum mechanics made its own creators uncomfortable.

Is the world quantum?

Published: November 2003

Characterizing Quantum Theory in Terms of Information-Theoretic Constraints

Rob Clifton, Jeffrey Bub & Hans Halvorson

Foundations of Physics **33**, 1561–1591(2003) | Cite this article

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Published: 22 October 2009

Information causality as a physical principle

Marcin Pawłowski, Tomasz Paterek, Dagomir Kaszlikowski, Valerio Scarani, Andreas Winter & Marek Żukowski

Nature **461**, 1101–1104(2009) | Cite this article

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Quantum nonlocality as an axiom

Sandu Popescu & Daniel Rohrlich

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Published: 20 February 2015

Almost quantum correlations

Miguel Navascués, Yelena Guryanova, Matty J. Hoban & Antonio Acín

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Almost-Quantum Correlations Violate the No-Restriction Hypothesis

Ana Belén Sainz, Yelena Guryanova, Antonio Acín, and Miguel Navascués

Phys. Rev. Lett. **120**, 200402 – Published 18 May 2018

2153 Accesses | 725 Citations | 26 Altmetric | Metrics

Our contributions

- We proposed a new physical principle
  - Based on a seminal theorem in epistemics
- We checked that Quantum Mechanics respects this principle
- We identified some other theories that do not respect it
  - In the process, we give a simple test for future theories

Quantum Correlations

- Correletions between observers of bipartite systems are nowdays <u>the only way</u> we proved that the world is not newtonian
  - Quantum computers still do not show *quantum advantage*

- *Entanglement* is the key resource to show such correlations

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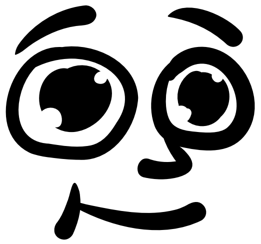
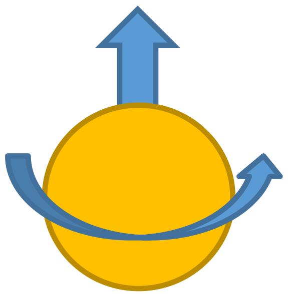
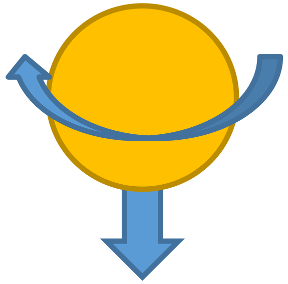




collapse of the wave function



Entanglement alone is not sufficient

You prepare a system, that when observed is either “UP - DOWN” or “DOWN - UP”, perfectly correlated.

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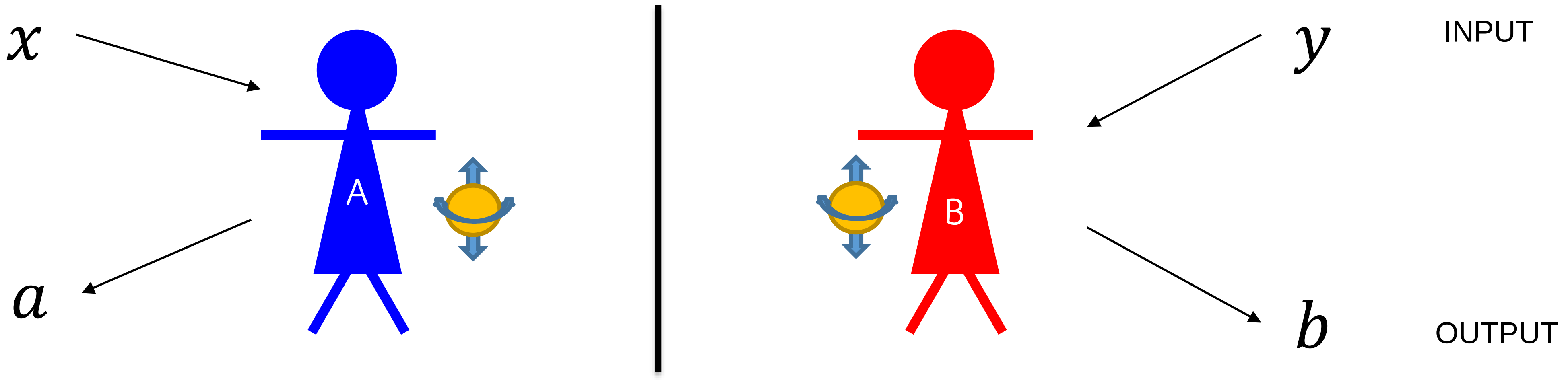
¿AND SO WHAT?

One needs an additional tool, discovered by physicist John Bell in the '60s, in order to rule out the **"hidden variable theories".**

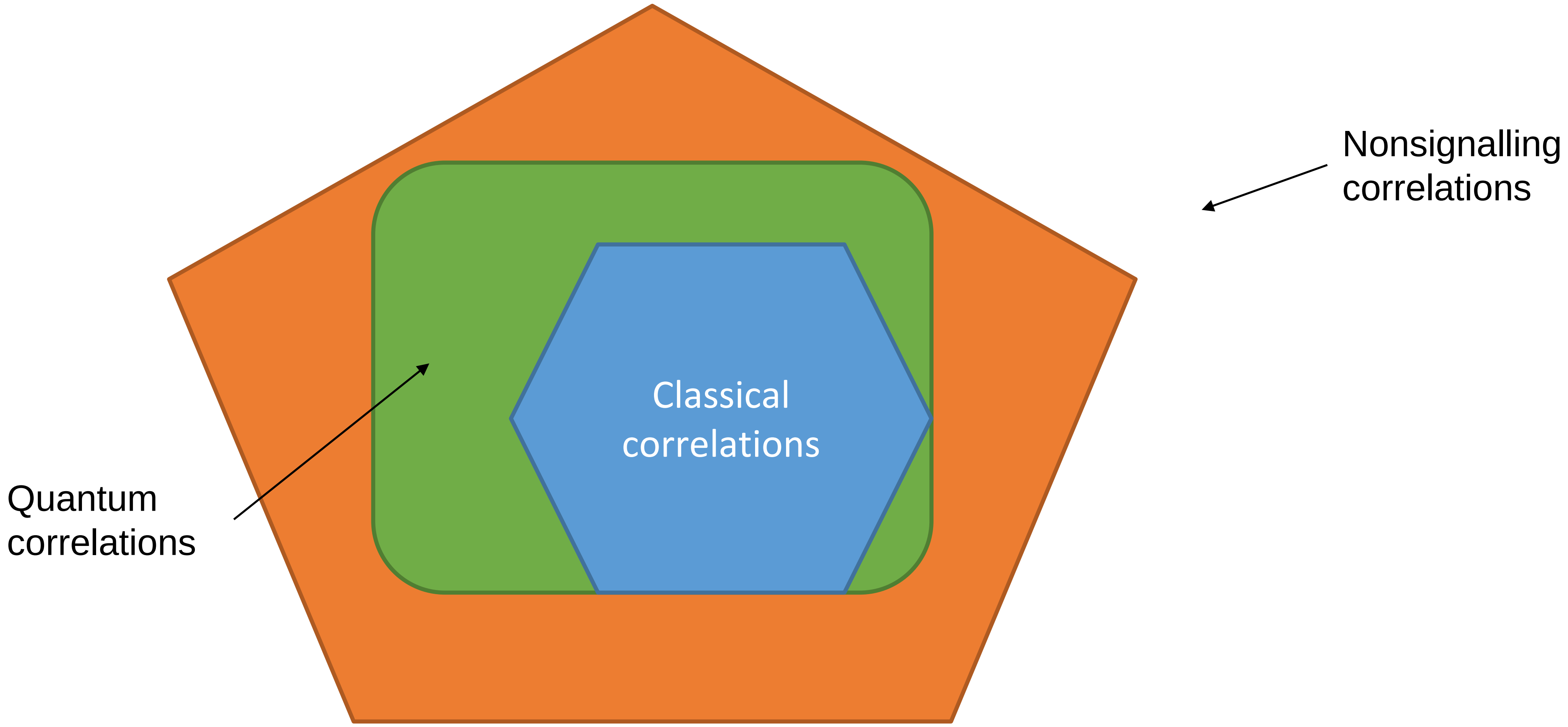
Games: creating correlations

- To demonstrate that a system is entangled, we analyze the consequences of its use as a resource.

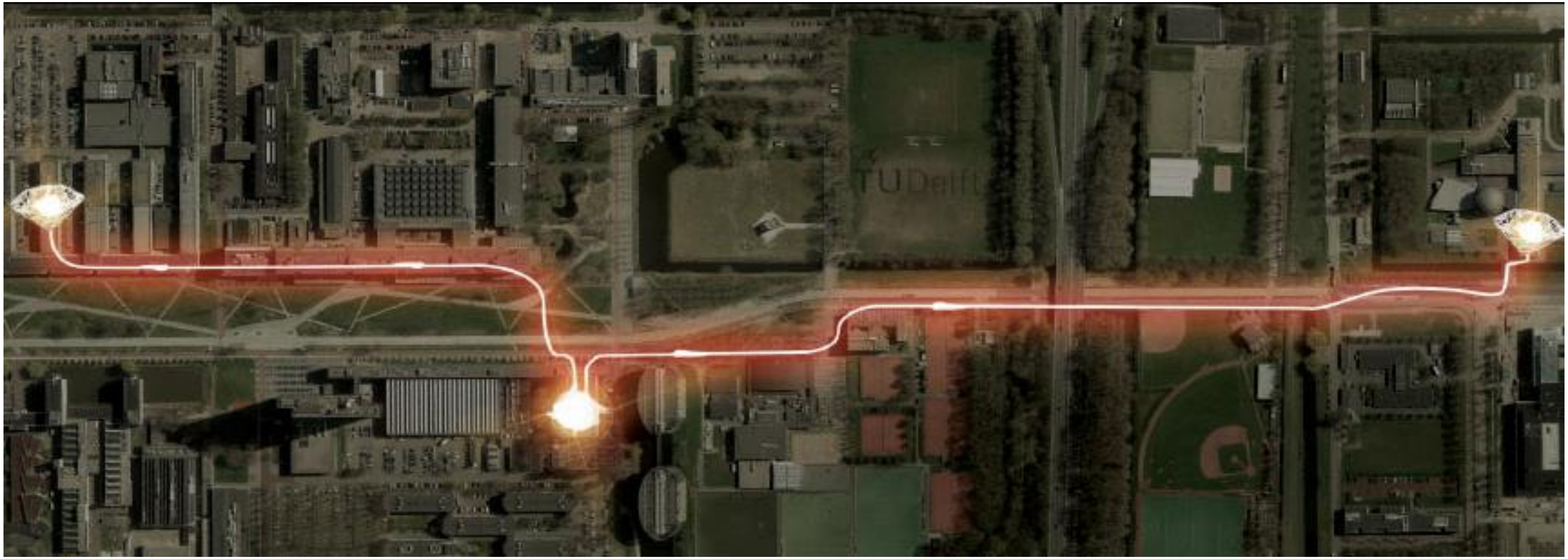


You get a correlation: $P(ab|xy)$

Hierarchy of correlations



Experiment in Delft (2015)



World isn’t classical!

- People with small quantum computers will be able to create quantum correlations $p(ab|xy)$.
- Applications:
    - Information security (quantum key distribution, device independence…)
    - Distributed computing
    - …
    - **Trading & finance** (e.g. *high frequency trading)*
    - **Decision making**

… And here begins our story.

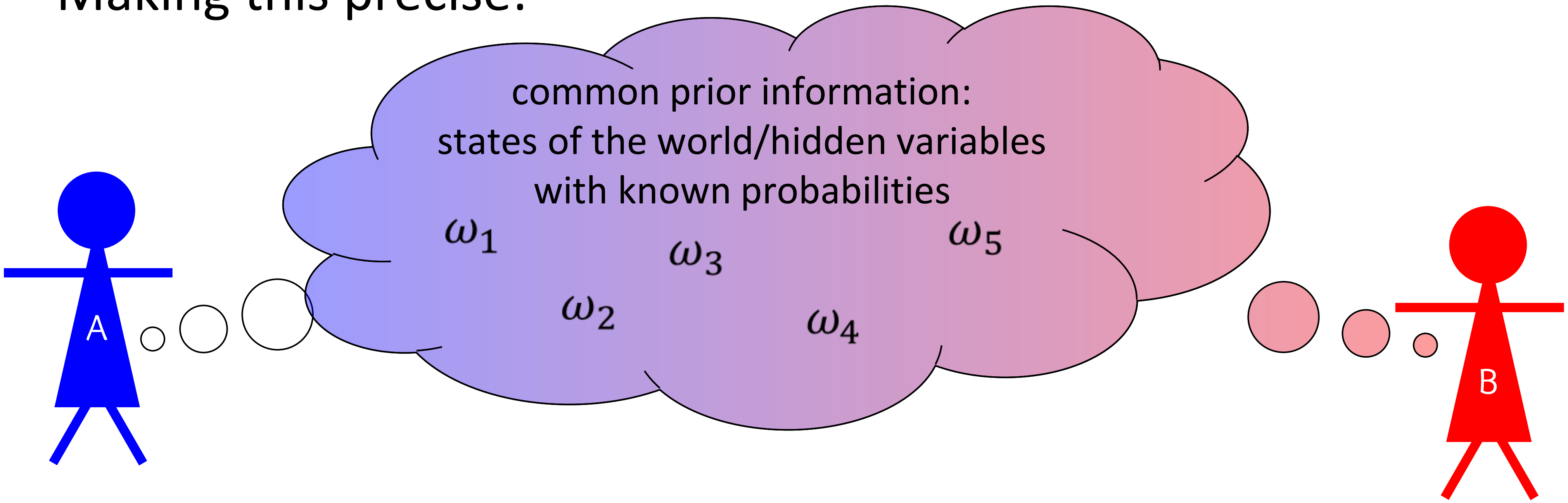
Aumann’s agreement theorem (1976)

- “Two agents who share a common prior cannot agree to disagree”

Aumann, *The Annals of Statistics*, **4** (6) 1236.

Aumann's agreement theorem (1976)

- "Two agents who share a common prior cannot agree to disagree"
- Making this precise:



Local model: $P(ab|xy) = \sum_{\boldsymbol{\omega}} \boldsymbol{p}(\boldsymbol{\omega}) P_A(a|x\boldsymbol{\omega}) P_B(b|y\boldsymbol{\omega})$

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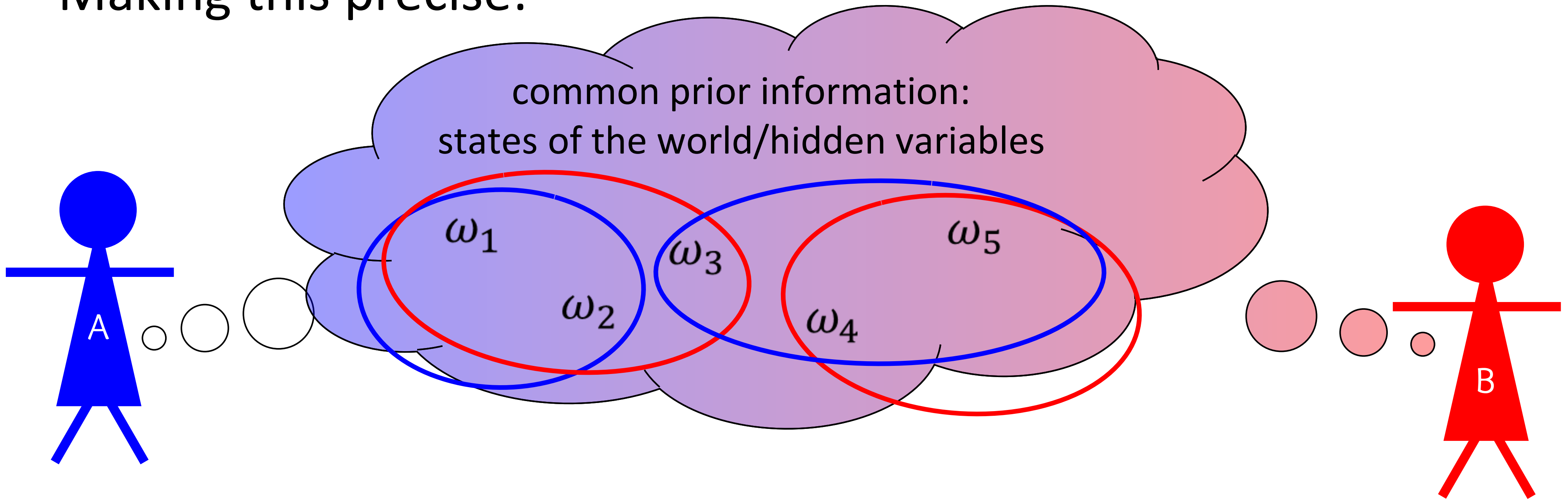

Alice and Bob divide the states of the world into different **partitions**
Know which partition element they're in

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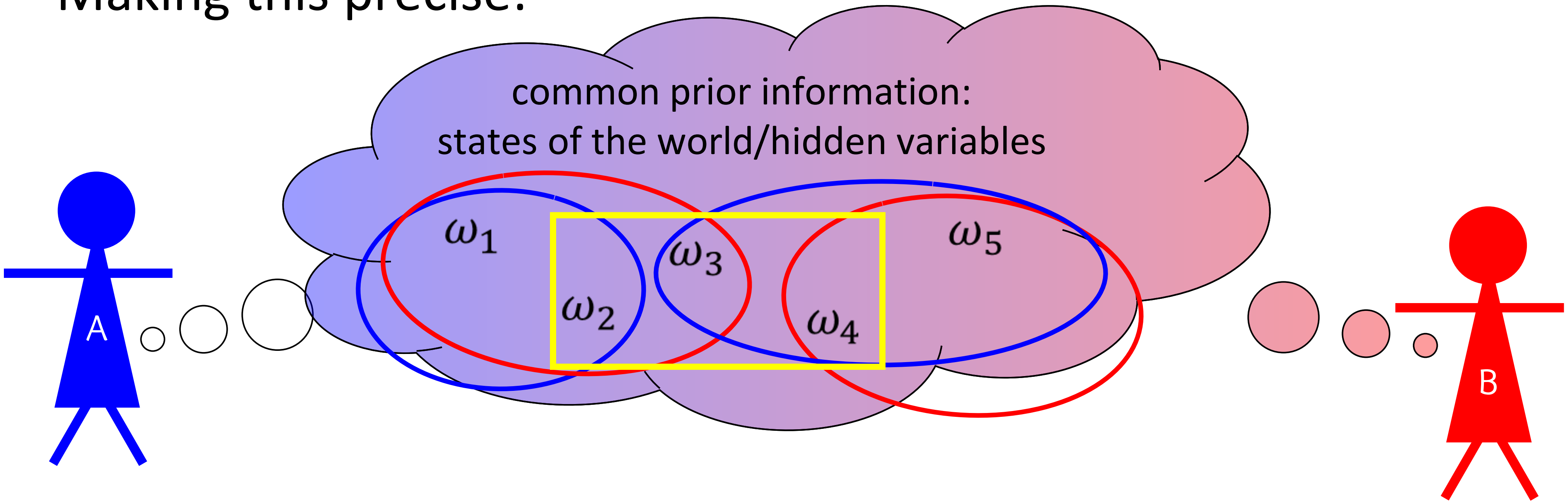

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Events of interest: sets of states of the world

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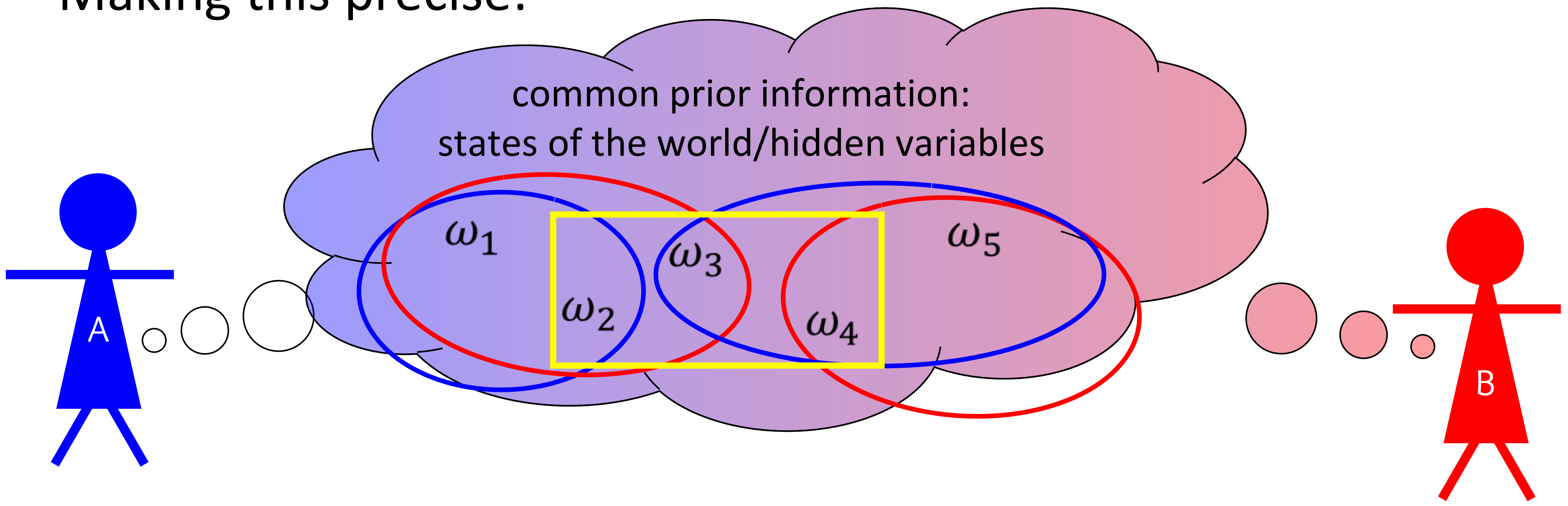

Alice and Bob divide the states of the world into different **partitions**
Events of interest: sets of states of the world
Work out conditional probabilities based on known partition element

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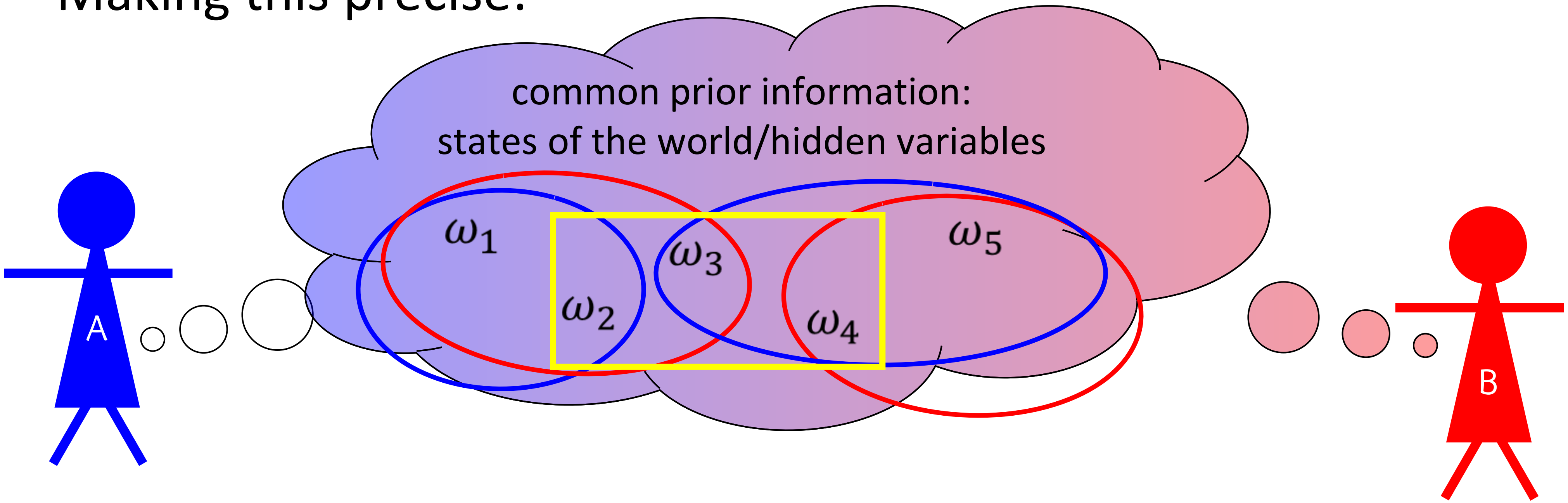
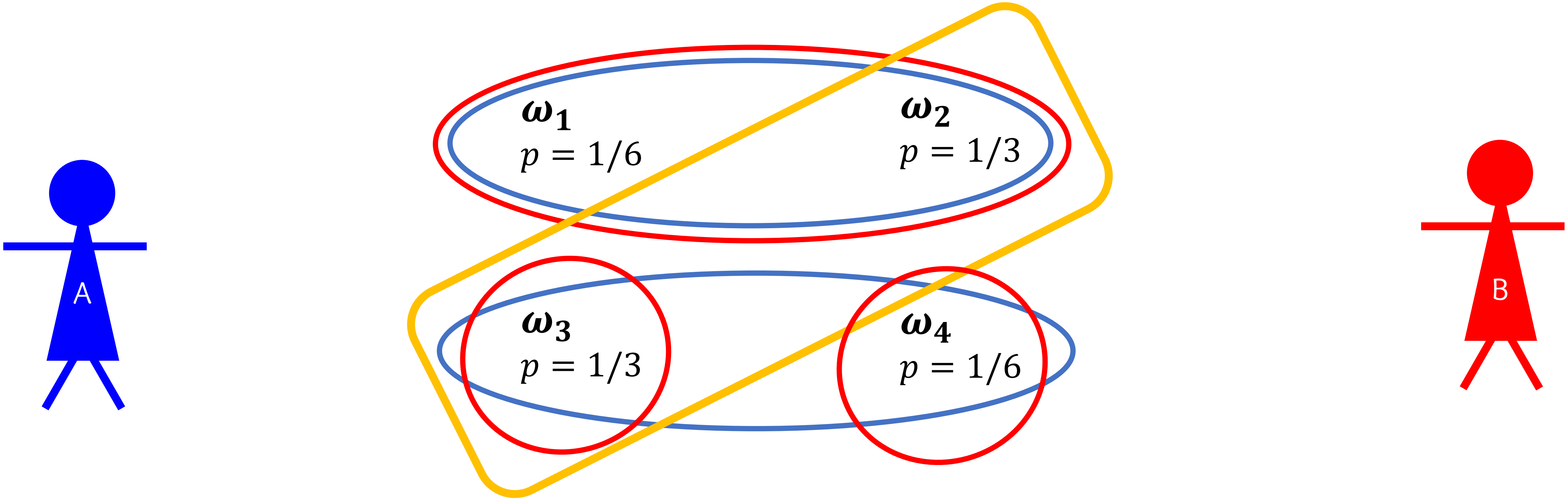


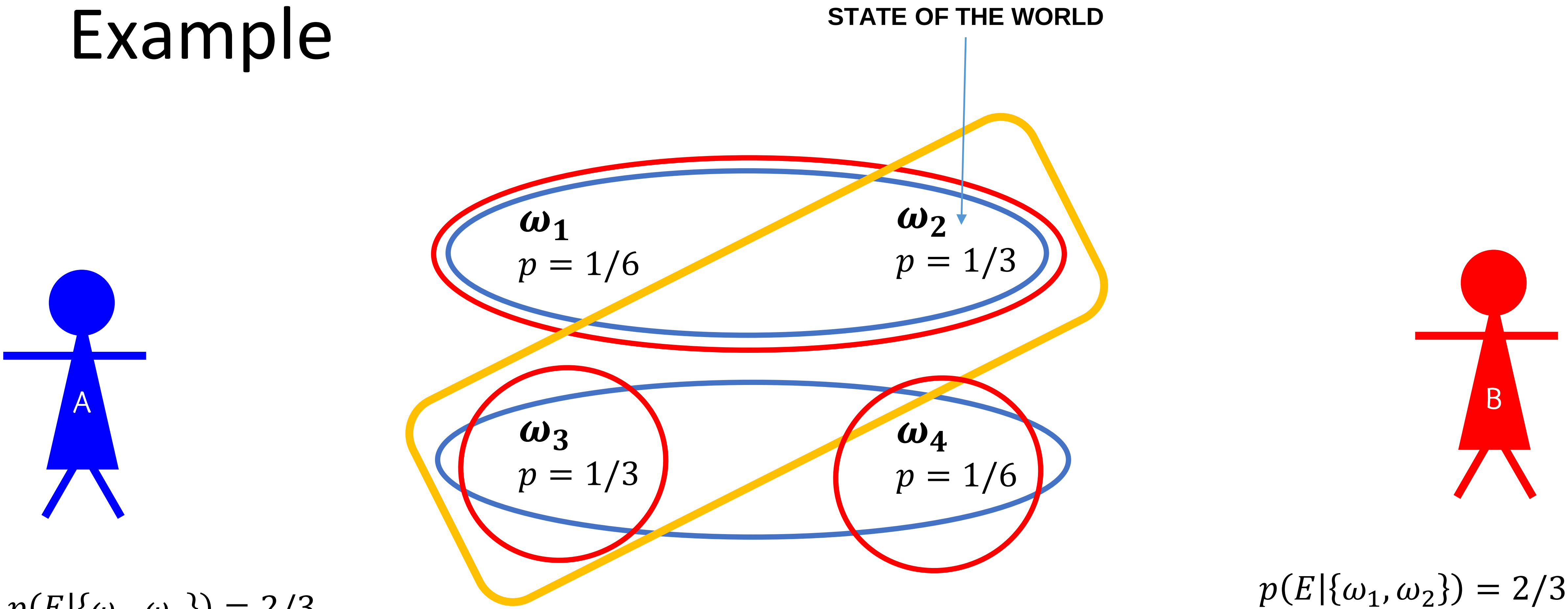
Work out conditional probabilities based on known partition element
Estimates are *common certainty* ⇒ they must be equal

Example



A
$\boldsymbol{\omega_1}$
$p = 1/6$
$\boldsymbol{\omega_2}$
$p = 1/3$
$\boldsymbol{\omega_3}$
$p = 1/3$
$\boldsymbol{\omega_4}$
$p = 1/6$
B

Example



$p(E|\{\omega_1, \omega_2\}) = 2/3$

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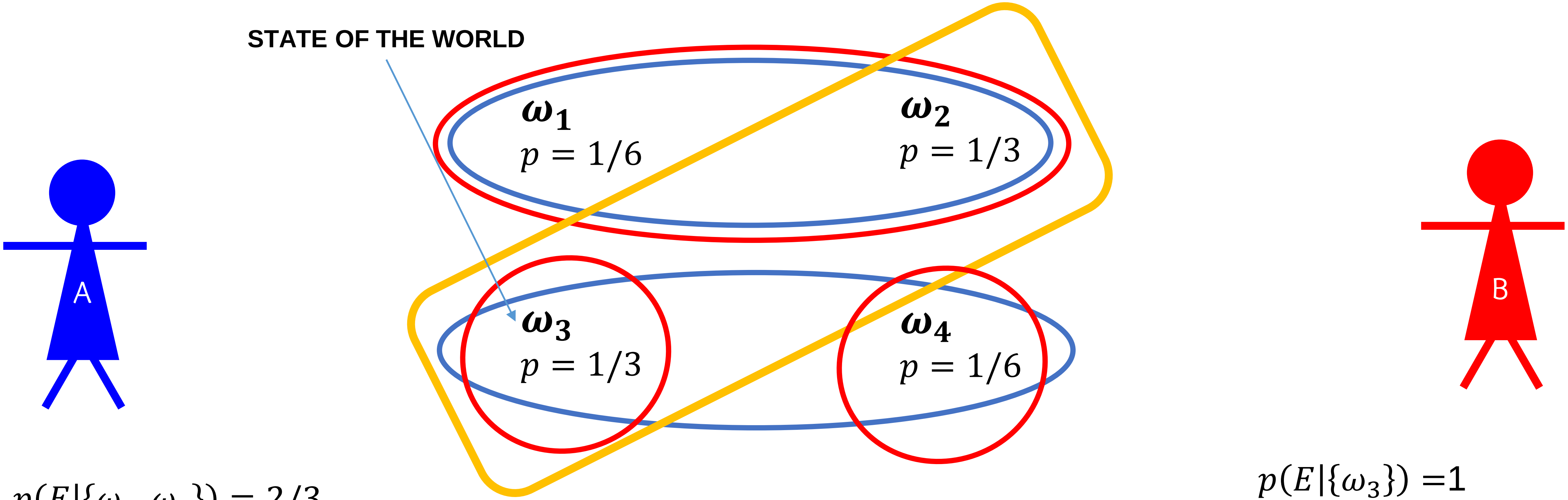
They agree, and it is common knowledge

Example

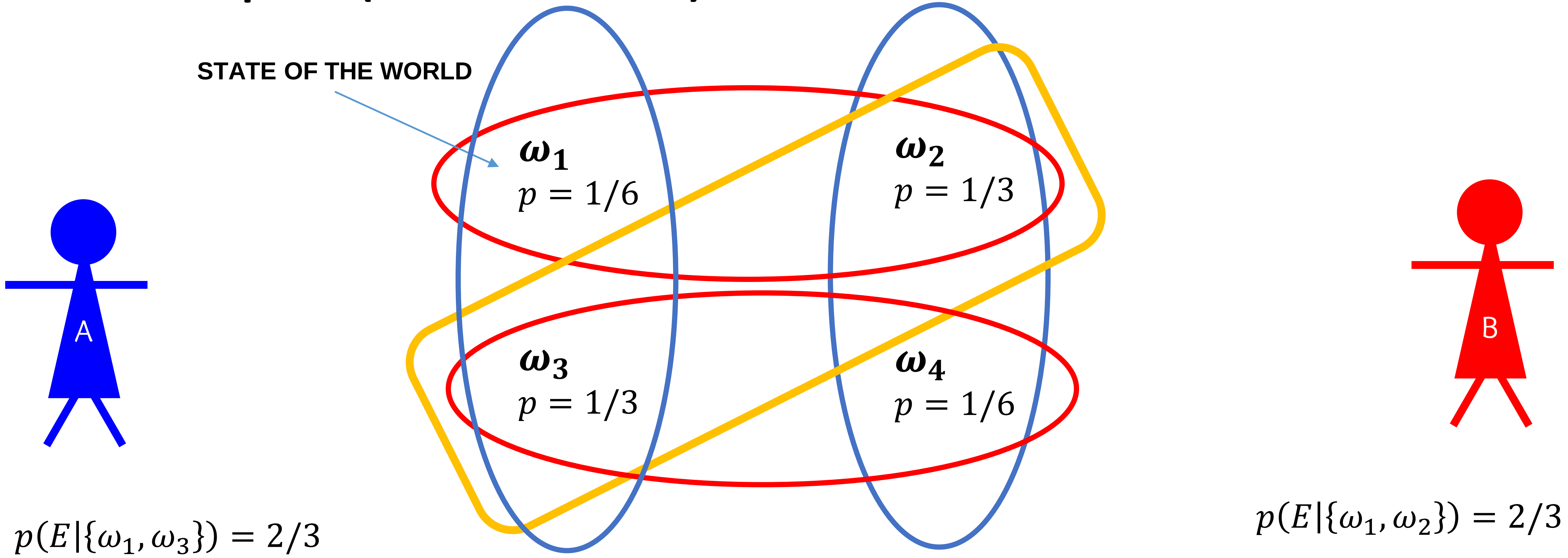


$p(E|\{\omega_3, \omega_4\}) = 2/3$

$p(E|\{\omega_3\}) = 1$

They DISagree, and it is NOT common knowledge

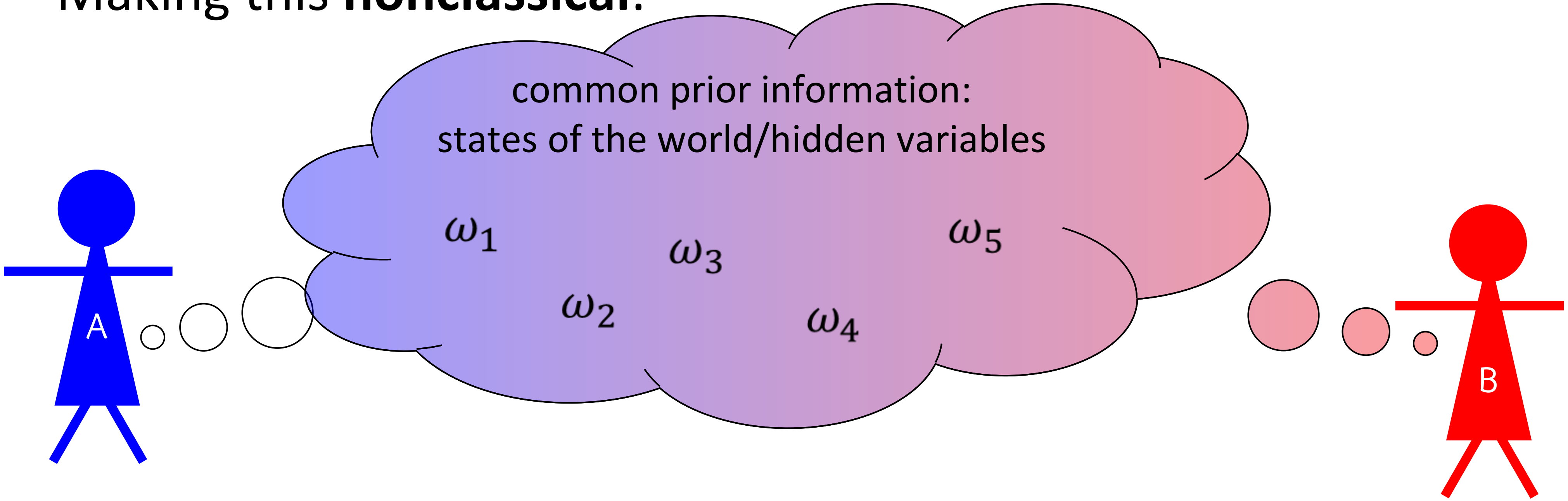
Example (less trivial)



The assignments are common knowledge, and therefore they must be equal, even if A&B reach the conclusion in different ways.

Aumann’s agreement theorem (1976)

- “Two agents who share a common prior cannot agree to disagree”
- Making this **nonclassical**:



Bell’s Theorem: there is no local model that can reproduce the predictions of QM
i.e. no hidden variables

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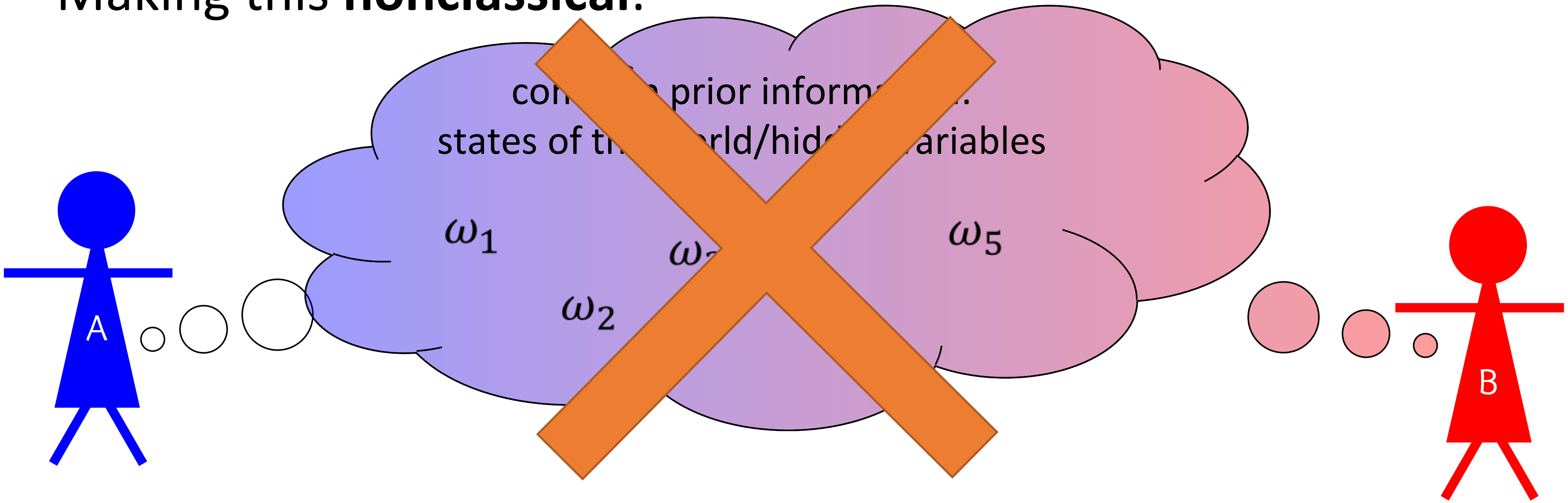

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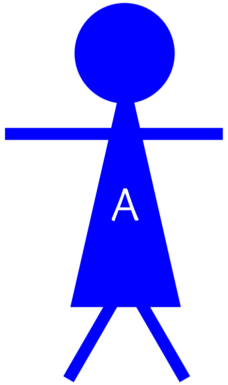
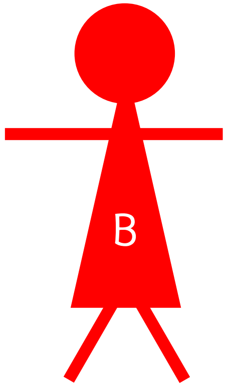


Bipartite probability distribution $\{p(ab|xy)\}$

- Not (necessarily) classical
- Nonsignalling: $\Sigma_a\, p(ab\,|\,xy) = \Sigma_a\, p(ab|x'y)$



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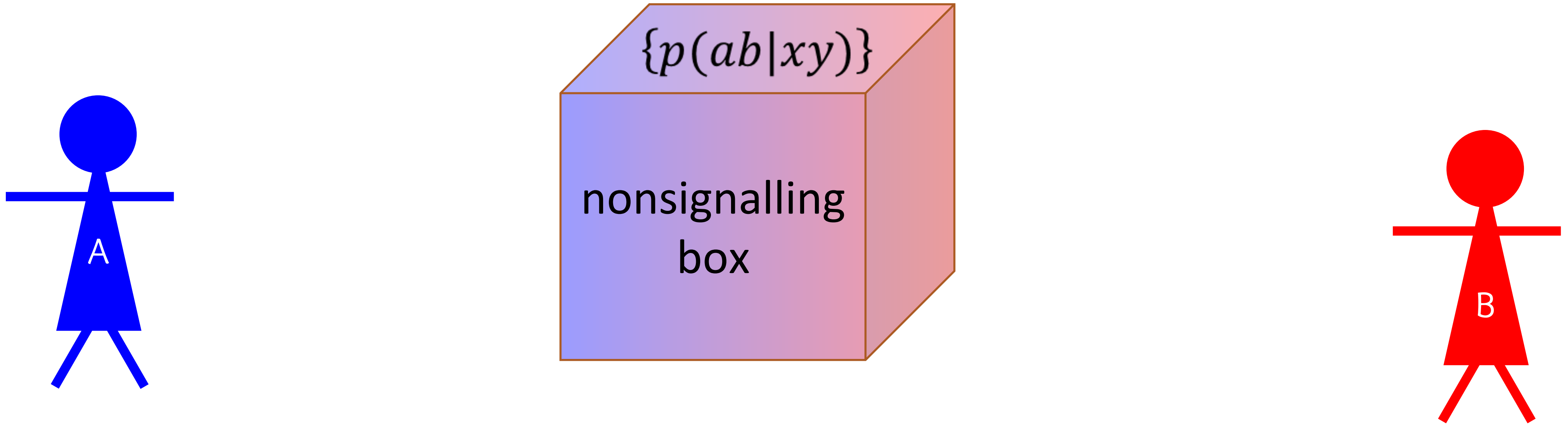

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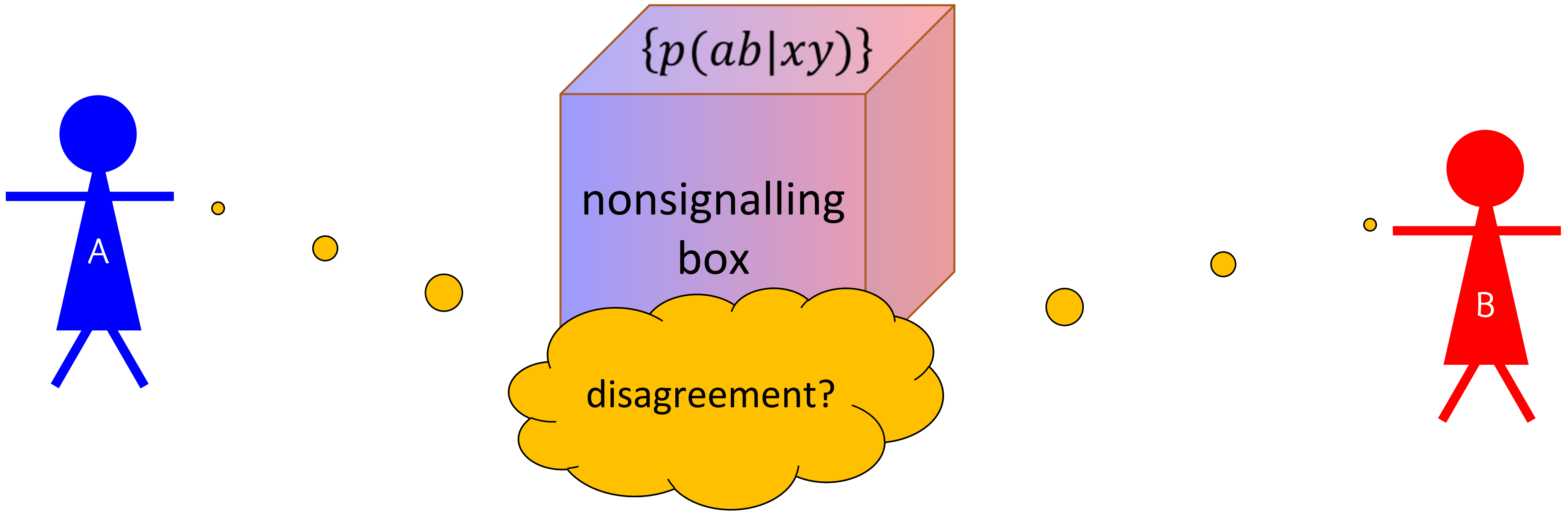
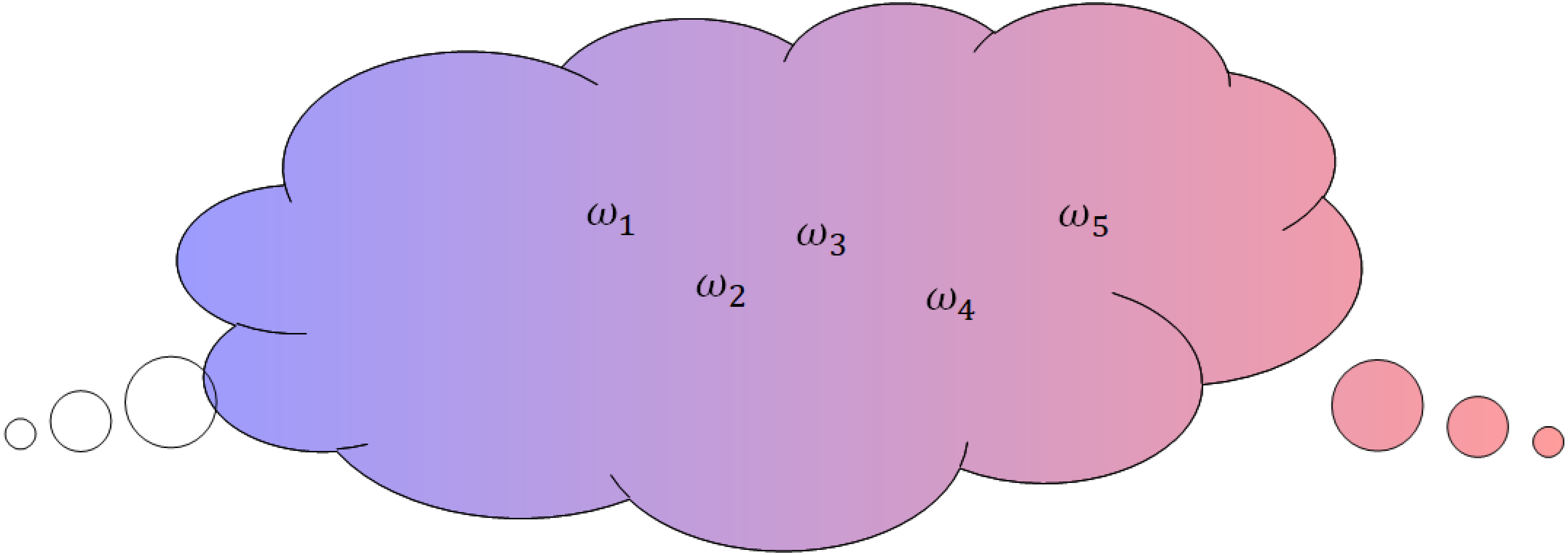


From LHVs to boxes and back again



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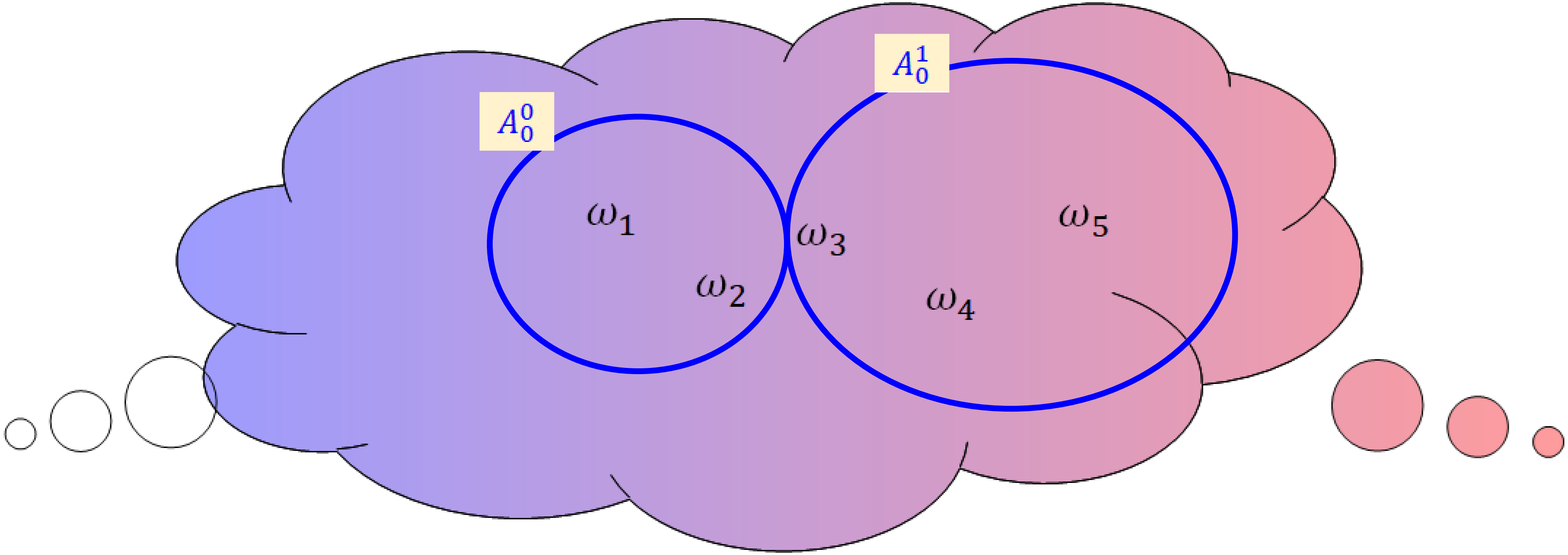
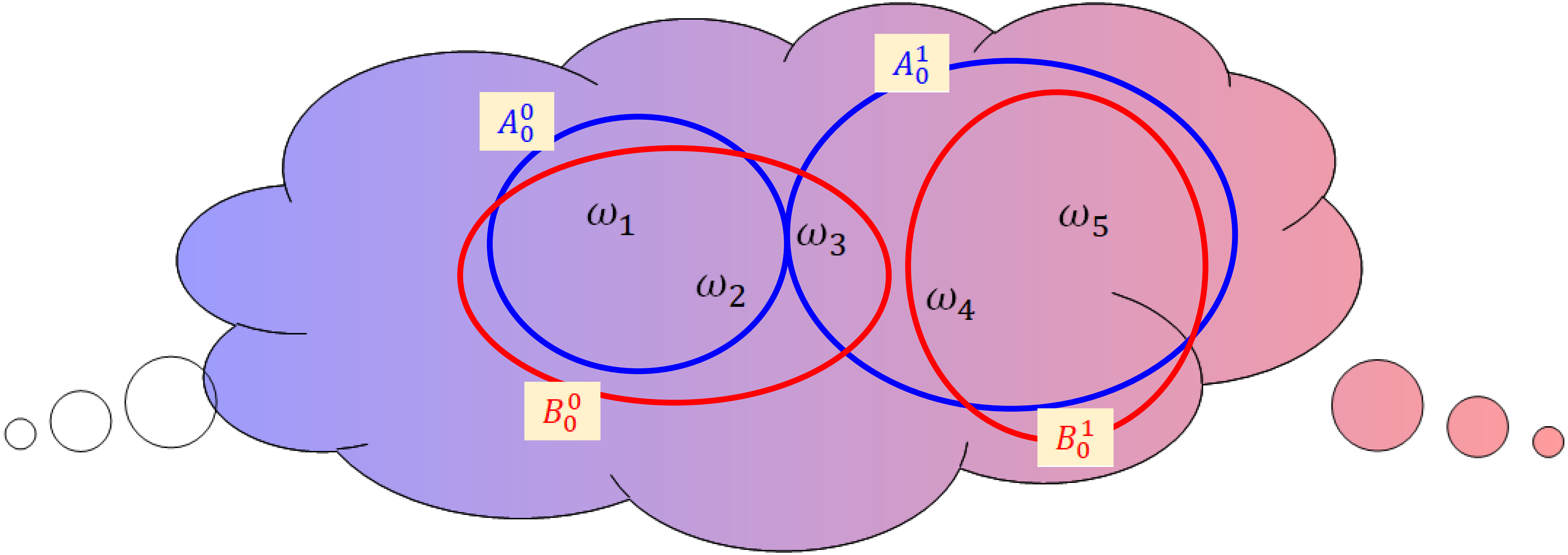


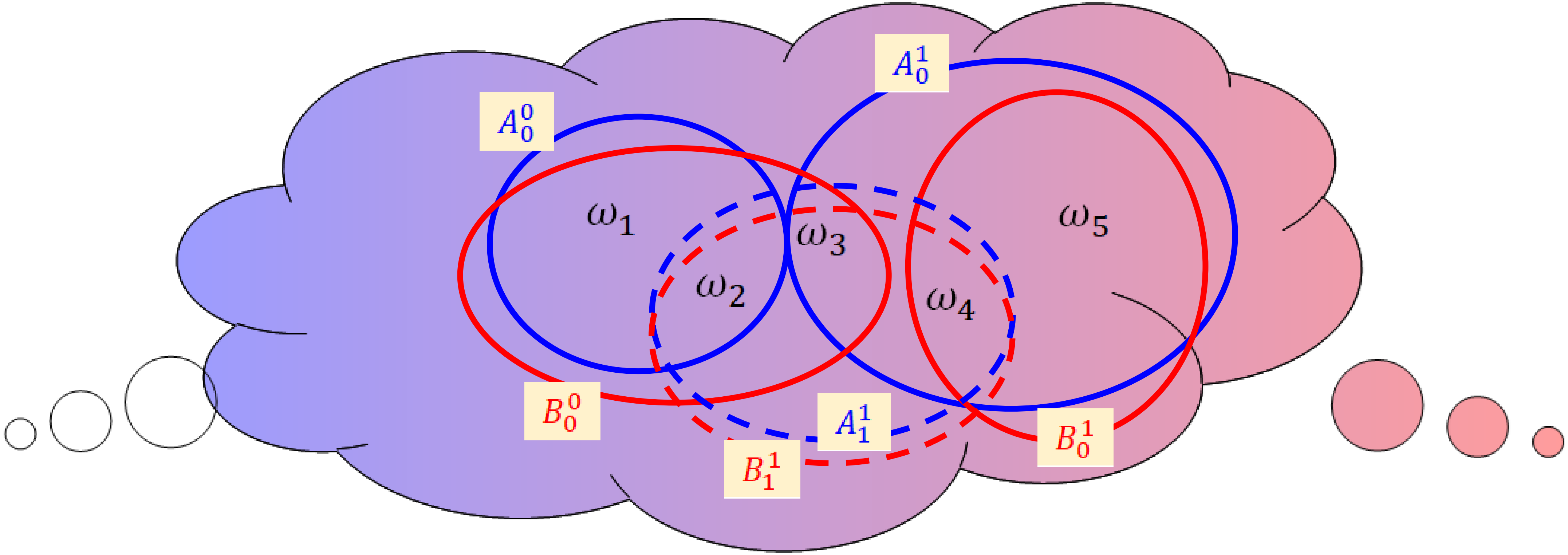
A_x^a = set of LHVs corresponding to output a given input x.

From LHVs to boxes and back again



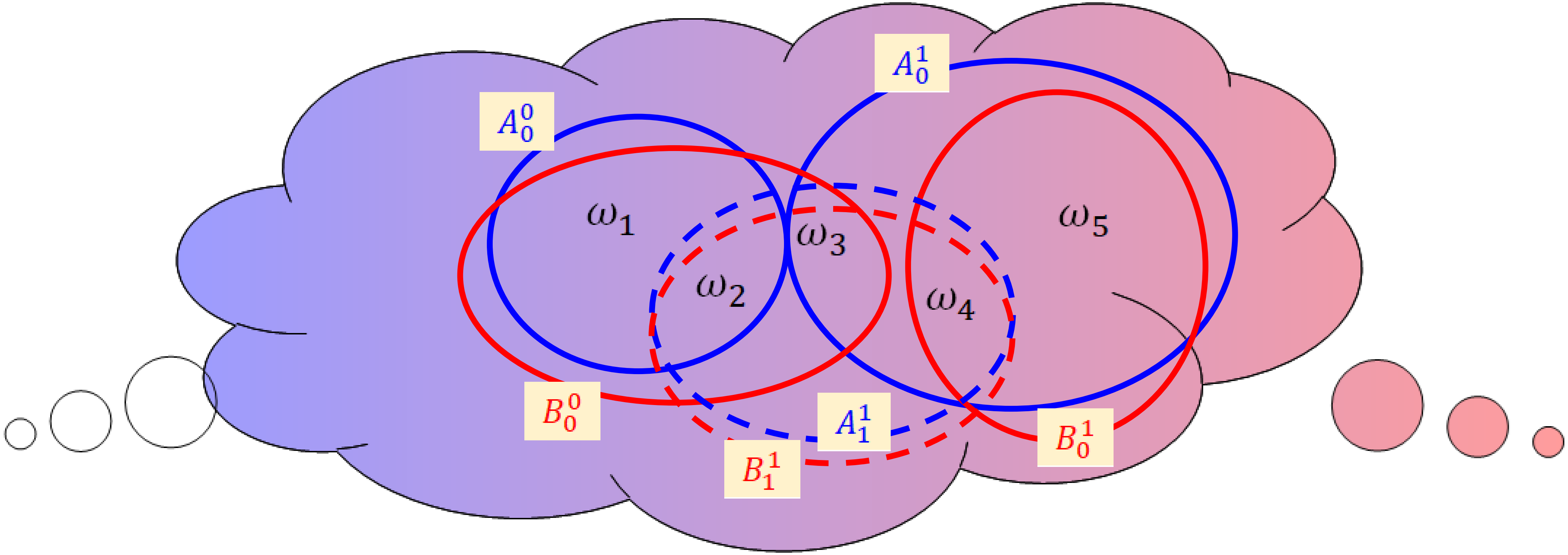
B_y^b = set of LHVs corresponding to output b given input y.

From LHVs to boxes and back again



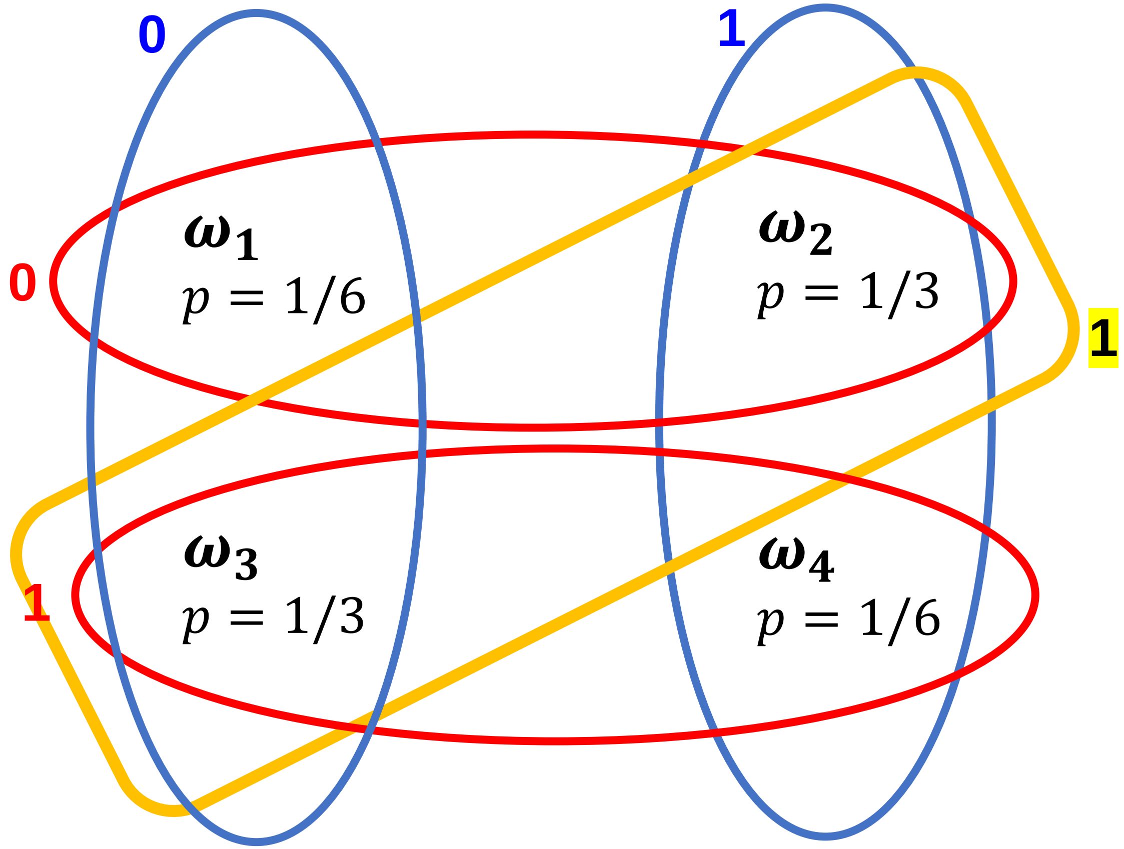
Perfectly correlated events: same set of LHVs (except prob zero)

From LHVs to boxes and back again

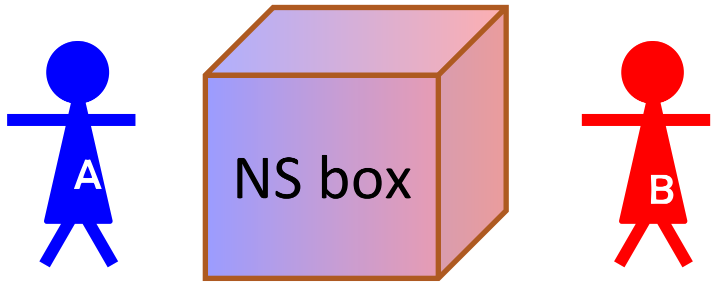


$$p(ab|xy) = P\left(A_x^a \cap B_y^b\right)$$

NS-box formalism

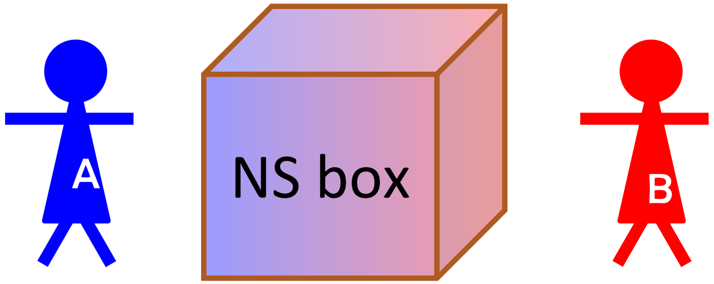


Input 0 to observe her partition, 1 to observe the Event



xy\ab	00	01	10	11
00	1/6	1/3	1/3	1/6
01	…	…	…	…
10	…	…	…	…
11	1/3	0	0	2/3

An operational interpretation



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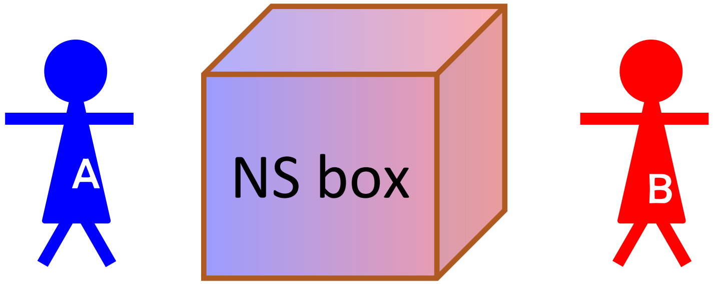
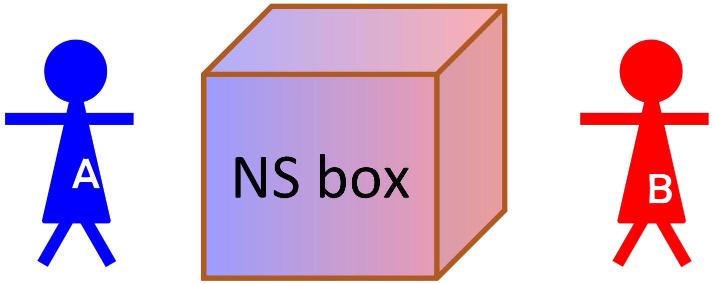


1. Measure $x, y = 0{,}0$, obtain $a, b = 0{,}0$ (but don't talk)

xy\ab	00	01	10	11
00				
01				
10				
11				

$p(ab|xy)$

An operational interpretation



1. Measure $x, y = 0{,}0$, obtain $a, b = 0{,}0$ (but don't talk)
2. Want to find out about each other's output 1 on input 1…

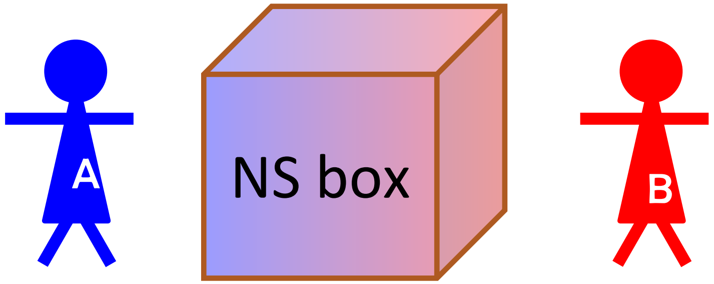
   … but they're constrained by nonlocality.

   Still, they try.

xy\ab	00	01	10	11
00				
01				
10				
11				

$p(ab|xy)$

An operational interpretation



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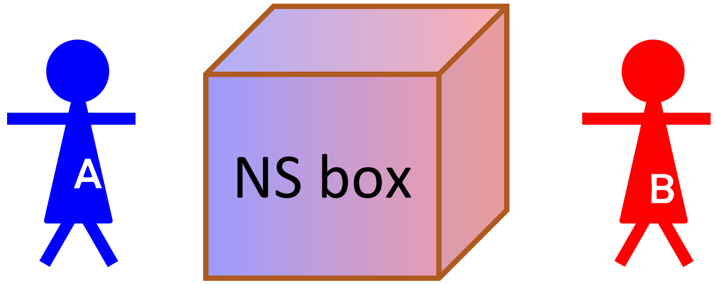
   Alice: $p(b = 1 | a = 0, x = 0, y = 1) =: q_A$

xy\ab	00	01	10	11
00				
01				
10				
11				

$p(ab|xy)$

An operational interpretation



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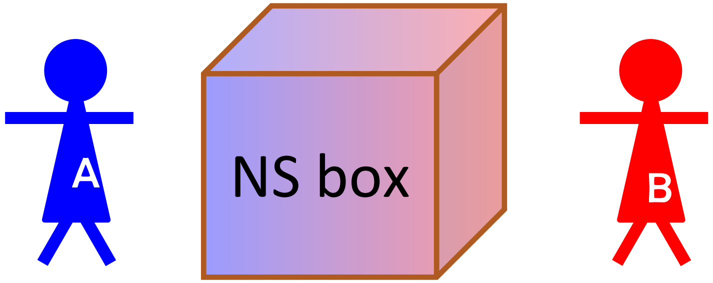
2. Want to find out about each other's output 1 on input 1:
   Alice: $p(b = 1|a = 0, x = 0, y = 1) =: q_A$
   Bob: $p(a = 1|b = 0, x = 1, y = 0) =: q_B$
3. Announce q_A, q_B

xy\ab	00	01	10	11
00				
01				
10				
11				

$p(ab|xy)$

An operational interpretation



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2. Want to find out about each other's output 1 on input 1:

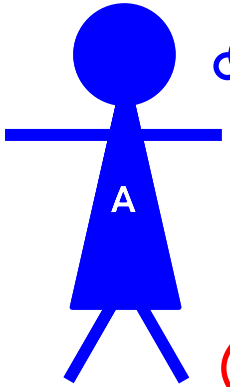
   Alice: $p(b = 1|a = 0, x = 0, y = 1) =: q_A$

   Bob: $p(a = 1|b = 0, x = 1, y = 0) =: q_B$
3. Announce q_A, q_B
4. Calculate a *sequence of sets* of input-output pairs in order to reach common certainty

xy\ab	00	01	10	11
00				
01				
10				
11				

$p(ab|xy)$

An operational interpretation

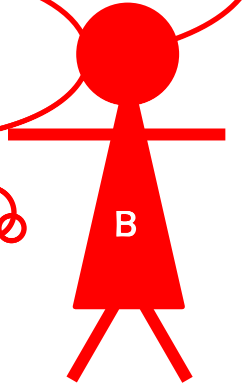


What output did Bob get?

$B_0 =$ {Outputs of Bob that lead him to announce the estimate q_B}

What output did Alice get?

$A_0 =$ {Outputs of Alice that lead her to announce the estimate q_A}



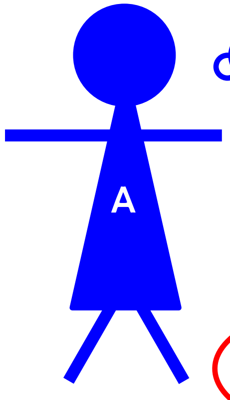
$$B_0 = \{(a, b, x, y): p(a = 1|b, x = 1, y = 0) = q_B\}$$

$$A_0 = \{(a, b, x, y): p(b = 1|a, x = 0, y = 1) = q_A\}$$

How to disagree

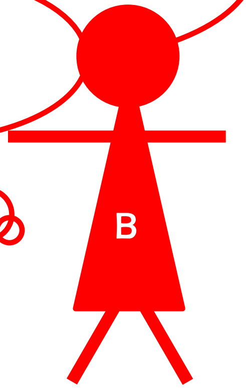
Can I be certain that Bob obtained something in B_0?

$A_1 = \{\text{Alice's outputs s.t. } B_0 \text{ is certain}\}$



Can I be certain that Alice obtained something in A_0?

$B_1 = \{\text{Bob's outputs s.t. } A_0 \text{ is certain}\}$



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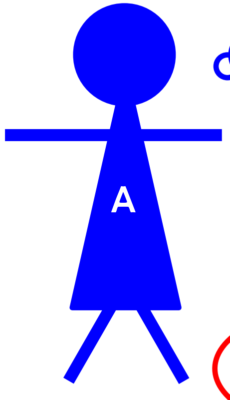
$$B_1 = \{(a,b,x,y): p(A_0|b,x=0,y=0) = 1\}$$

An operational interpretation

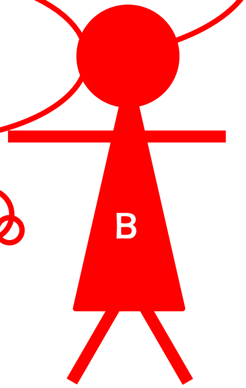
Can I be certain that Bob obtained something in $\boldsymbol{B_{n-1}}$?

A_n = {Alice's outputs s.t. B_{n-1} is certain}



Can I be certain that Alice obtained something in $\boldsymbol{A_0}$?

B_n = {Bob's outputs s.t. A_{n-1} is certain}



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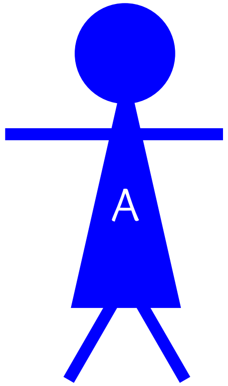
…

$$A_0 = \{(a,b,x,y):\ p(b=1|a,x=0,y=1) = q_A\}$$

$$B_1 = \{(a,b,x,y):\ p(A_0|b,x=0,y=0) = 1\}$$

…

Common certainty of disagreement

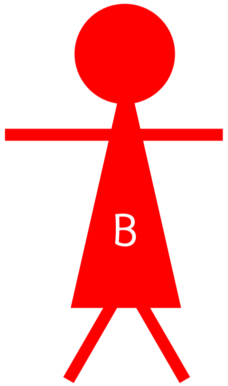


Common certainty of disagreement

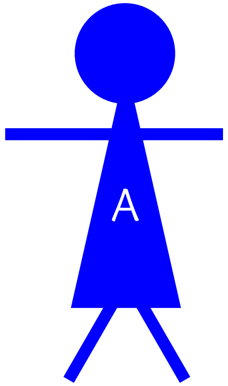
$\Leftrightarrow$

$q_A \neq q_B$ &

$\forall n, (0,0,0,0) \in A_n \cap B_n$



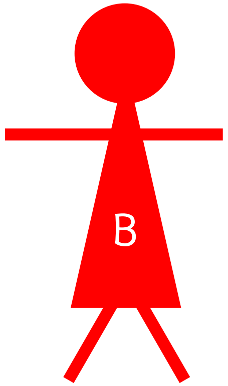
Common certainty of disagreement



Common certainty of disagreement

$\Leftrightarrow$

Aumann:
impossible classically



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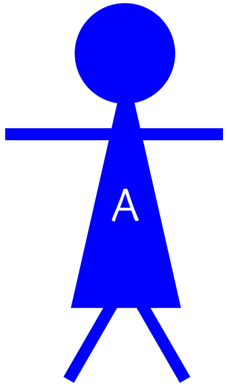
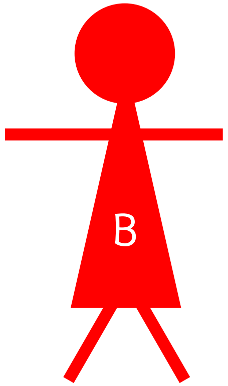


Common certainty of disagreement

$\Leftrightarrow$

Can it arise in quantum/nonsignalling settings?



Characterising common certainty of disagreement

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Theorem: Yes!

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00	r	0	0	$1-r$
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with $r > 0, s - u \neq r - t$ (otherwise classical)

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Proof: zeros from perfect correlations and sets A_n, B_n.
Rest from normalisation & nonsignalling constraints.

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$(0,0,0,0) \in B_1 \Rightarrow p(a = 0|b = 0, x = 0, y = 0) = 1$

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$(0,0,0,0) \in B_1 \Rightarrow p(a = 0|b = 0, x = 0, y = 0) = 1$
$\Rightarrow p(a = 1|b = 0, x = 0, y = 0) = 0$

Characterising common certainty of disagreement

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Similarly for A_1.

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Assume first $A_0 = \{a = 0\};\ B_0 = \{b = 0\}$ (other cases will reduce to this)
Similarly for A_1.
NS constraints & normalisation. $q_A \neq q_B \Leftrightarrow s - u \neq r - t$.

Characterising common certainty of disagreement

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Proof: Reverse implication quite easy.

Characterising common certainty of disagreement

Can it arise in quantum settings?
Theorem: No.

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with $r > 0$, $s - u \neq r - t$ (otherwise classical)

Proof:

1. Theorem by Tsirelson: if the box is quantum, there is a vectorial representation of the system whose inner products relate to the elements of the box.
2. One checks that the parameters r,s,t,u above create a quantum box with *s-u=r-t* (i.e., a classical box and therefore obeying Aumann's original theorem).

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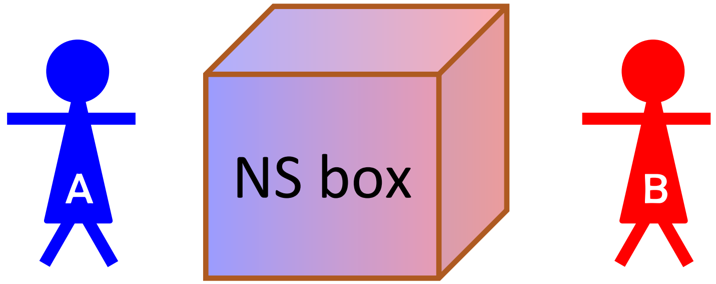
Fully general: distributions of more inputs and outputs reduce to 2x2 by local transformations.
So if 2x2 can't be quantum, nor can larger ones.

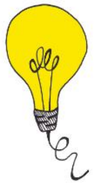
Implications

- First, we are closer to understanding why QM is a successful theory.
  - We also provide a simple test for new physical theories (use the box!)
- The quantum internet will soon be a reality: agents will use it to trade, take decisions, communicate securely, perform calculations...
- We are closer to showing that all of this <u>will make sense:</u> modelling the applications of new technologies will be sound.

Where to?

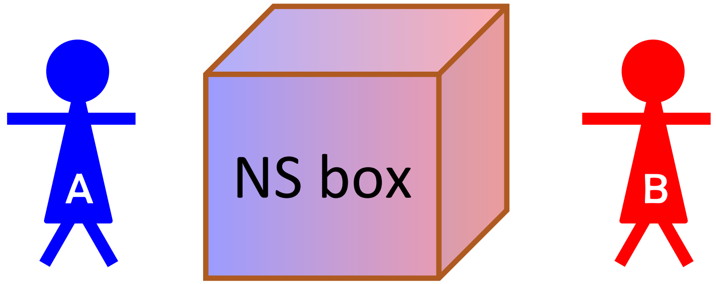


- Agreement as a requirement for physical theories of nature
- Approximate notions of disagreement?
- Application to distributed computation



- Further connections between epistemics and quantum information?

Where to?



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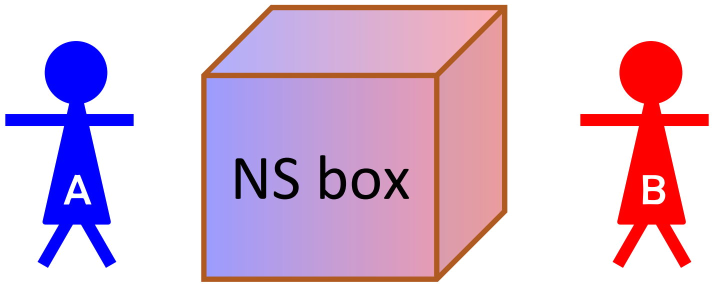
- **Approximate notions of disagreement?**
- Application to distributed computation

- Further connections between epistemics and quantum information?

New model (work in progress!)



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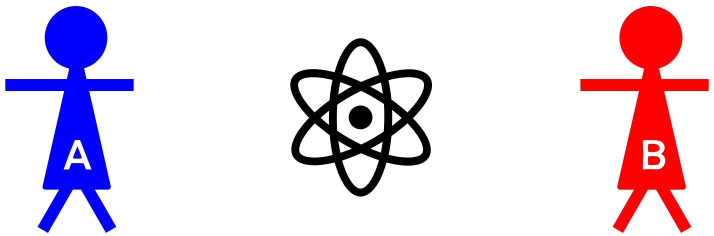

- There is no box, no communication
- A single system observed in sequence by two agents
- Models the system like in standard quantum computing (and not with an approximation as we have seen before)

- Moreover: we compare the quantum scenario with a classical scenario by Hellman, which considers *degrees* of disagreement:
  - δ-distance in prior distributions $\Longrightarrow$ δ-disagreement

Recap

- Can Alice & Bob disagree?
  - local ✗
  - quantum ✗
  - post-quantum (nonsignaling) ✓
- Consequences for computer scientists:
  we can have fun researching the quantum internet!
- Next step: approximate version, with single system



Nat. Comm. 12, 7021

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