Towards Extensible Symbolic Formal Methods

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In this way it can scale up verification efforts to handle large systems used in industrial practice.

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$(\Sigma, E \cup B)$ has the finite variant property (FVP) iff any term t has a finite set of most general variants.

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For example, $x > y \equiv y > x$ has the single unifier $\{x \mapsto y\}$ modulo $\mathcal{N}_{+,>}$.

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Example of Variant-Based Satisfiability

Consider the quantifier-free formula:

$$head(l) > head(l') = \top \wedge head(l) > 1{+}1{+}1 = \bot \wedge \{(1{+}1); nil\} \subseteq \{l, l', \emptyset\} \neq tt$$

in the composition of: (i) Presburger arithmetic $\mathcal{N}_{+,>}$, (ii) the parameterized theory of lists $\mathcal{L}[X]$, and (iii) the parameterized theory of hereditarily finite (HF) sets $\mathcal{H}[Y]$. These three theories and their composition are FVP and have decidable satisfiability.

To decide satisfiability we:

1. first solve the sytem of equations $head(l) > head(l') = \top \wedge head(l) > 1 + 1 + 1 = \bot$ modulo the composed theory. There are six constructor unifiers. The first is: $\alpha = \{l \mapsto (1 + 1 + 1); l_1, l' \mapsto (1 + 1); l_2\}$.
2. This shows that the formula is satisfiable, because $\{(1 + 1); nil\} \subseteq \{(1 + 1 + 1); l_1, (1 + 1); l_2, \emptyset\} \neq tt$, is irreducible by the equations for $\subseteq$ modulo AGU.

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Note that narrowing happens at two levels:

- with rules R modulo $E \cup B$ to perform symbolic transitions
- with oriented equations E modulo B to compute $E \cup B$-unifiers by folding variant narrowing.

The Maude-NPA Crypto Protocol Analyzer

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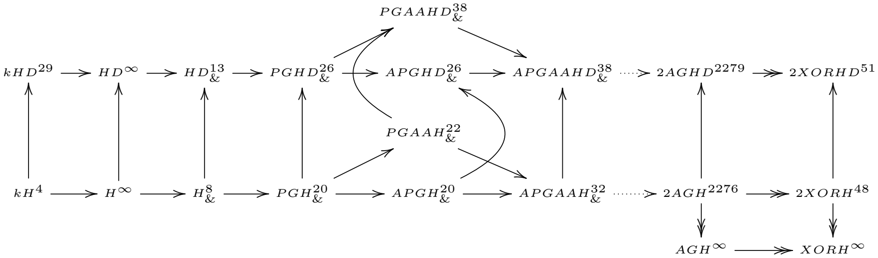

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eq N ; M ; [K1, crit(M1)] [K2, crit(M2)] PS |= ex? = false .
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