Analog computers and Schrödinger's cats

A pedestrian introduction to Quantum Computers

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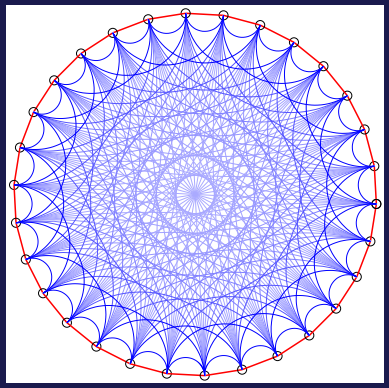


Facultad de Informática, UCM, Madrid. May 7, 2015.

What shall we talk about?

Simulation is the sincerest form of flattery

We're surrounded by computers, we only have to look

Always bet so that losing is the best outcome

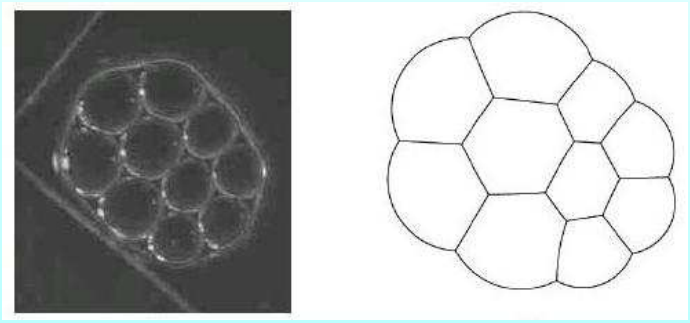
The best path is to take all paths

Solving problems requires some untidiness

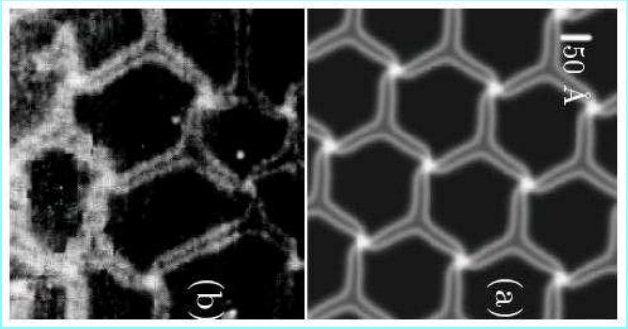
Can we learn physics studying computer science?

Physicists love to simulate

MthK protein (PDB: 1LNQ),
Bacterial Ca^{2+}-gated binding K channel
Courtesy Agata Kranjc, SISSA



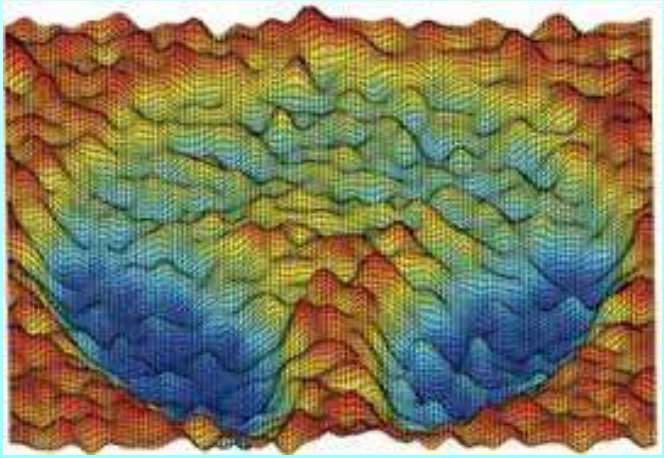
Foam configuration simulations,
[J. Phys. C: Cond. Matt. **16**, 4165 (2004)]
Courtesy Dolores Alonso, Trinity College



Pt(111) surface reconstruction,
[see also PRB **67**, 205418 (2003)]
Courtesy Raghani Pushpa, SISSA

It's hard to simulate!

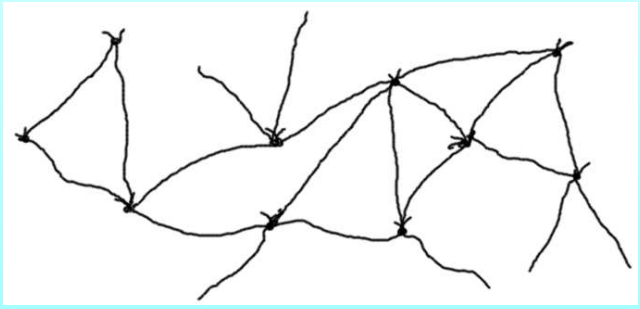
- Very often, Nature **optimizes**.
- Simulating optimization can be **pretty hard**.
- Target functions can have very complicated **landscapes**.



What if we can't?

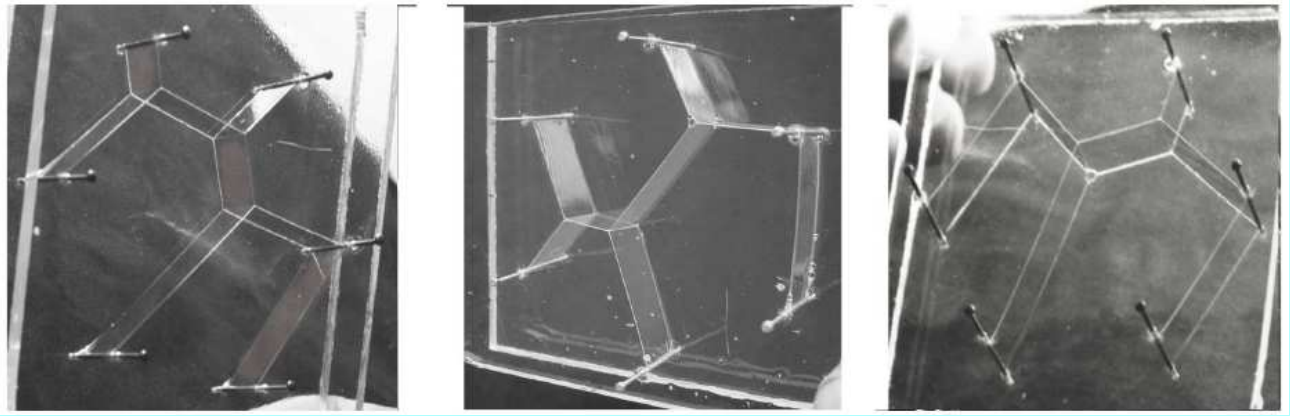
- What if simulation is out of our reach?
- We can profit from that to devise **analog computers**!

Spaghetti computer



Stringy computer

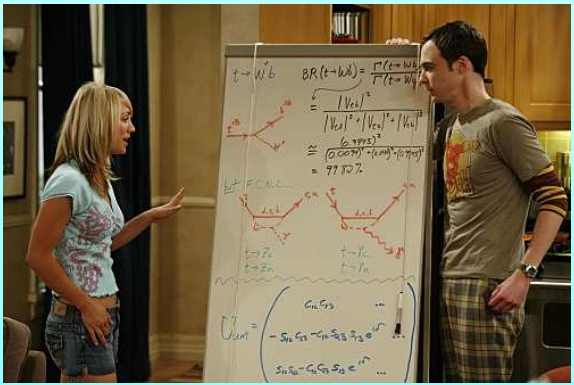
Classical Analog Computers



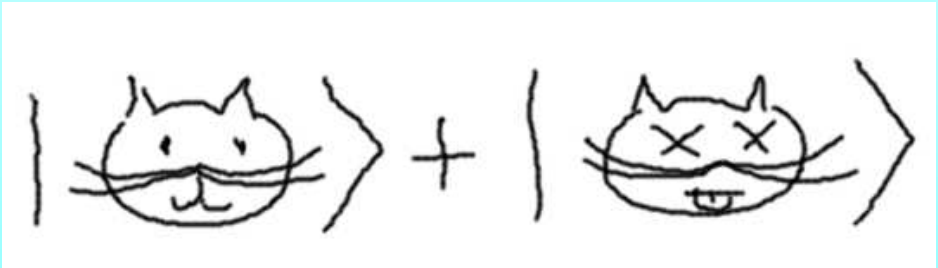
Experiment by Dutta and coworkers, `ArXiv: 0806.1340`.

Nature need not be a Turing machine

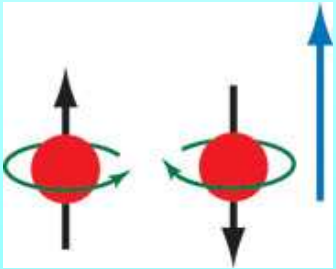
- Can we simulate efficiently Nature in a Turing machine?
- Classical mechanics: **maybe**.
- Quantum mechanics: **no way**.



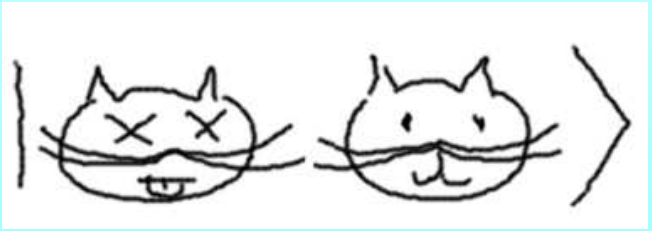
Schrödinger's cat

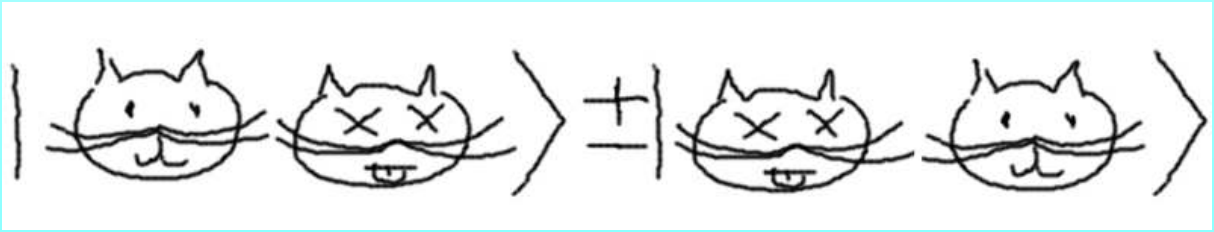


Schrödinger's cat



Entanglement





EPR experiment

- Prepare an entangled state $|+-\rangle - |-+\rangle$.
- Measure **any** component on one spin, you get:

- Entanglement entropy: how much information you lose forgetting one part.

Many-Cat Physics

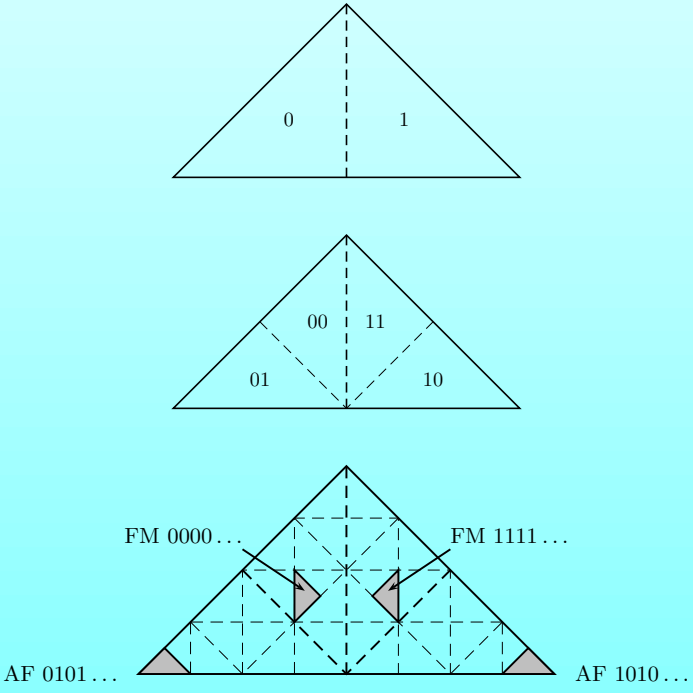
- Also known as **many-body physics**.
- A quantum pure state is a mapping

$$\psi : \{0,1\}^N \mapsto \mathbb{C}$$

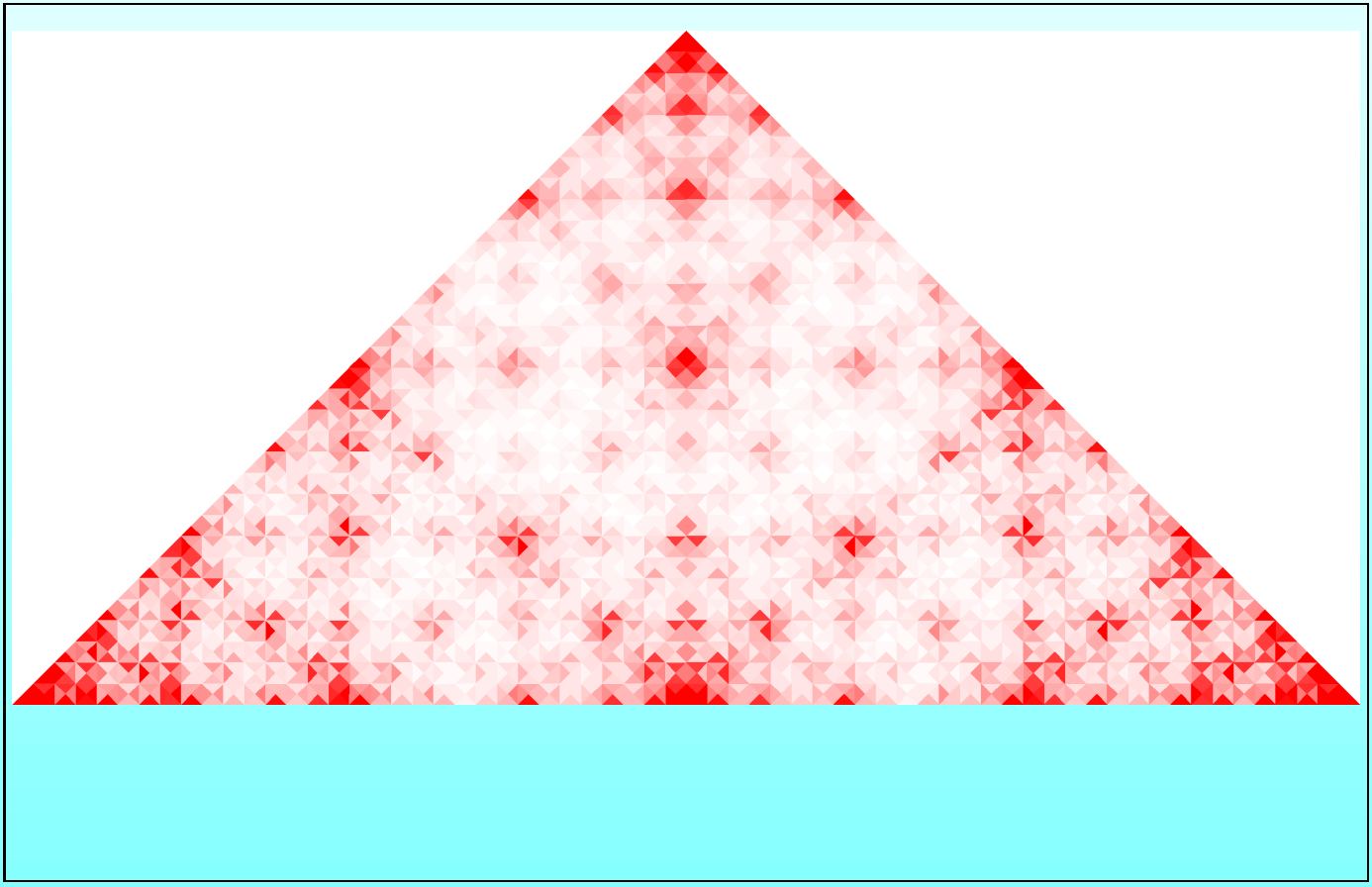
- Example: $\alpha\,|010101\rangle + \beta\,|101010\rangle$.
- 2^N **components**... a lot!
- They are not epistemological, they are ontological!!!

Qubistic view

- How to represent graphically a pure state?
- QUBISM, developed by us (2012).

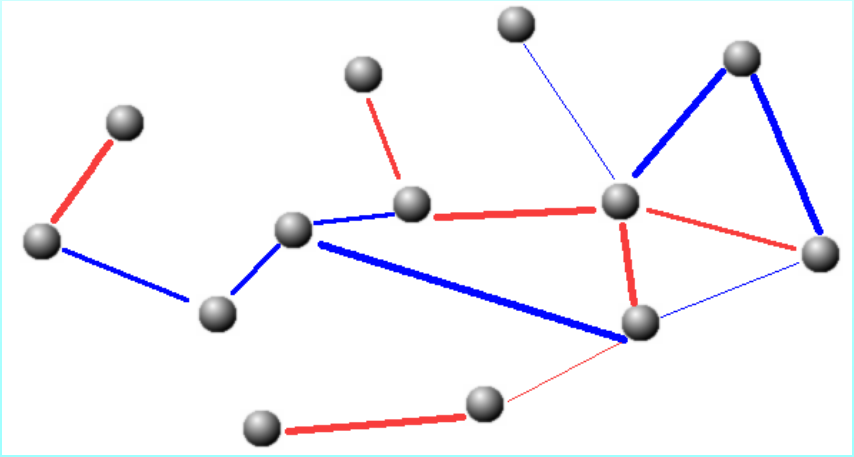


Qubistic view



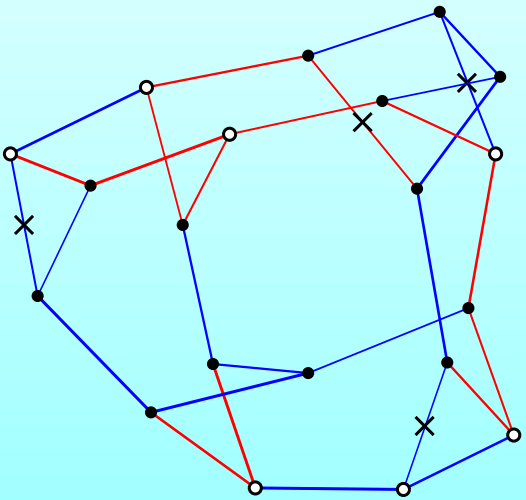
Solving all your life problems

- **Nodes:** life aims, **Links:** constraints.



Also known as **Spin-Glass Problem**

Solving all your life problems



- **Minimize frustration**

Adiabatic Quantum Computation

$$H_F = \sum_{\langle i,j \rangle} J_{ij} S_i^z S_j^z + \sum_i h_i S_i^z$$

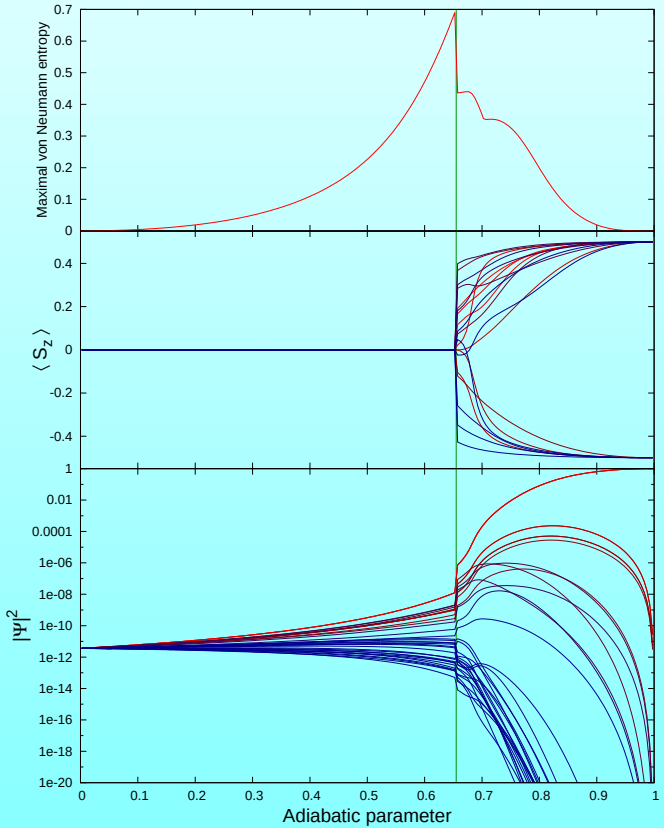
$$H_0 = \sum_i S_i^x$$

$$H(t) = (1-t)H_F + tH_O$$

- For $t = 0$, $|\Psi\rangle = |\rightarrow\rangle^{\otimes N}$.
- For $t = 1$, $|\Psi\rangle$ is the solution to our problem!!!

What happens in the middle?

Complexity of the State



The Unfolding Hypothesis

Moreover: complexity typically **explodes**, a Quantum Phase Transition.

Problems studied with AQC

- Exact cover.
- Ramsey numbers.
- Factoring.
- Unsorted search.

— Main experimental problem: the **GAP**.

— Speed limit, to success probability larger than $1 - \epsilon$,

$$\left|\frac{dH}{dt}\right| / (\Delta E)^2 < \epsilon$$

Simulating AQC

- What makes Quantum Computation Special?
- **Simulated Thermal Annealing.**

Energy target function, E, finite temperature: $\mathsf{p}(\mathsf{X}) \sim \mathsf{exp}(-\mathsf{E}/\mathsf{T})$.

- **Simulated Quantum Annealing.**

The same, but with P replicas, joined by springs.

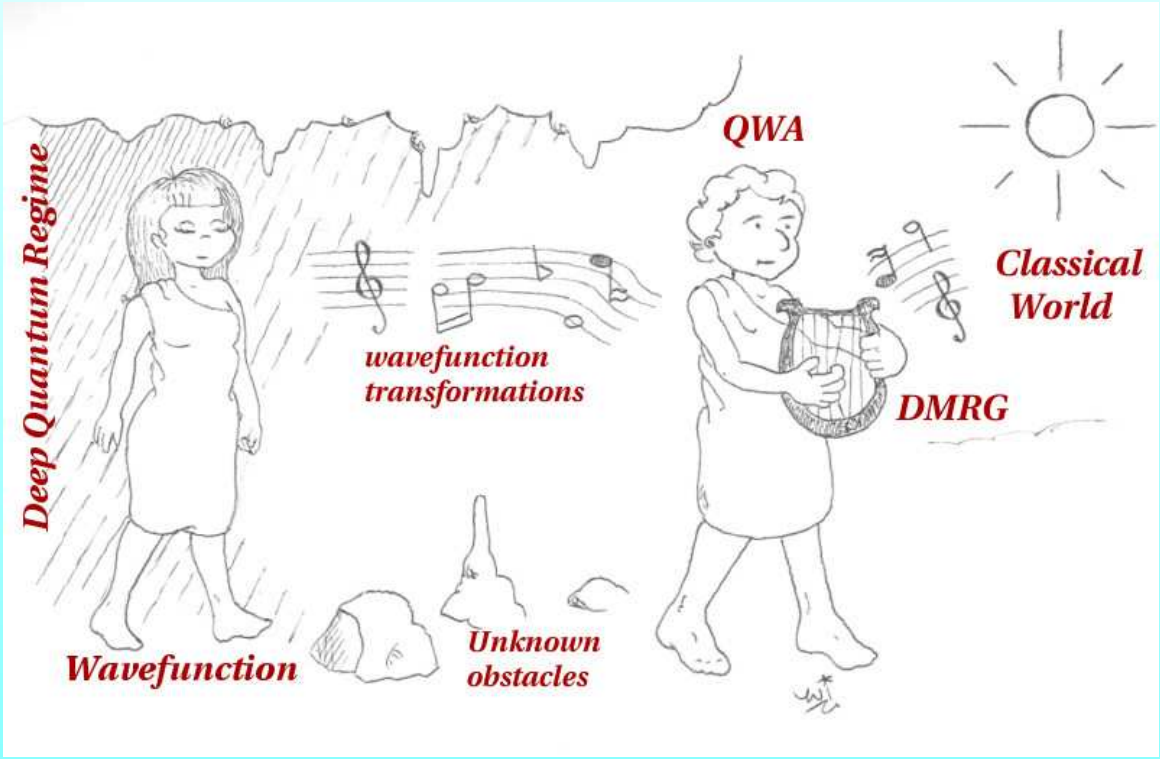
Matrix Product States

Efficient way to **store** pure states, $\psi : \{0,1\}^N \mapsto \mathbb{C}$.

$$\psi(s_1, \cdots, s_N) = \mathrm{Tr}\left(A_1^{s_1} A_2^{s_2} A_3^{s_3} \cdots A_N^{s_N}\right)$$

- $A_k^{\pm}$ are $2N$ matrices of dimension $m \times m$.
- Somehow, A are similar to finite-automata transition matrices.
- The dimension m is related to **entanglement**.
- Quantum Wavefunction Annealing: simulate AQC using MPS!
- Bottleneck of QWA: entanglement.

Quantum Mythology



Physical Predictions of P$\neq$ NP?

- Can we learn physics by doing computer science?
- Nature need not be a Turing machine... but our computers are!
- Thus, no simulation of AQC will solve **NP**-complete problems in polynomial time.
- Thus, our QWA simulation time must scale fast for them.
- Thus, we **must** encounter a quantum phase transition in our path!!

Will Quantum Computation Succeed?

- AQC depends on the existence of a gap.
- Gaps are typically found at Quantum Phase Transitions.
- Computer Science predicts Quantum Phase Transitions...
- YET, Nature is hard to simulate!!!!!!!!
- So... we don't know.

Thank you for your Attention!

- Visit our bar: `http://mononoke.fisfun.uned.es/jrlaguna`

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